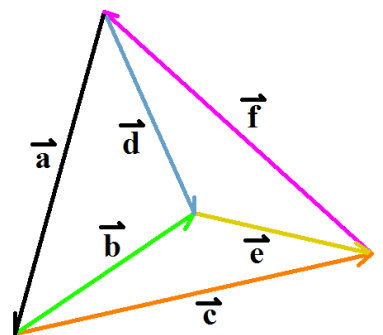


## Math 43 Midterm 2 Review

In addition to the following review questions, you must be able to solve any of the questions from the 3D Lines & Planes handout.

Some questions below will require the use of a calculator.

- [1] Consider the vectors  $\vec{f} = 2\vec{j} - 3\vec{k}$  and  $\vec{g} = -\vec{i} - 3\vec{j} + 4\vec{k}$ .
- [a] Find the angle between  $\vec{f}$  and  $\vec{g}$ . (Your answer should be in radians, rounded to 2 decimal places.)
  - [b] Find a unit vector perpendicular to both  $\vec{f}$  and  $\vec{g}$ . (Do **NOT** use decimal approximations.)
  - [c] If the terminal point of  $\vec{g}$  is  $(-7, 4, -8)$ , find the initial point.
  - [d] If  $\vec{h} = a\vec{i} + b\vec{j} - 5\vec{k}$  is parallel to  $\vec{g}$ , find the values of  $a$  and  $b$ .
  - [e] If  $\vec{e} = 7\vec{i} + c\vec{j} - 5\vec{k}$  is perpendicular to  $\vec{g}$ , find the value of  $c$ .
- [2] Let  $P$  be the point  $(-5, -2, 3)$ . Let  $Q$  be the point  $(3, 2, -1)$ . Let  $R$  be the point  $(-3, 4, -2)$ .  
Let  $\vec{u}$  be the vector with initial point  $R$  and terminal point  $Q$ .  
Let  $\vec{w}$  be the vector with initial point  $P$  and terminal point  $R$ .  
Let  $\vec{t} = 3\vec{i} - \vec{k}$ .
- [a] In which octant is  $R$ ?
  - [b] If you start at point  $P$ , move 2 units down, 4 units back, and 6 units to the right, find the co-ordinates of your ending point.
  - [c] Write  $\vec{u}$  in component form.
  - [d] Write  $\vec{w}$  as a linear combination of  $\vec{i}$ ,  $\vec{j}$  and  $\vec{k}$ .
  - [e] Find the magnitude of  $\vec{w}$ . (Do **NOT** use decimal approximations.)
  - [f] Find a unit vector in the opposite direction as  $\vec{w}$ . (Do **NOT** use decimal approximations.)
  - [g] Find a vector of magnitude 6 in the same direction as  $\vec{u}$ . (Do **NOT** use decimal approximations.)
  - [h] If  $\|\vec{v}\| = 3$ , and the angle between  $\vec{u}$  and  $\vec{v}$  is 2 radians, find  $\vec{u} \cdot \vec{v}$ . (Round your answer to 2 decimal places.)
  - [i] If  $\|\vec{v}\| = 3$ , and the angle between  $\vec{u}$  and  $\vec{v}$  is 2 radians, find the magnitude of  $\vec{u} \times \vec{v}$ . (Round your answer to 2 decimal places.)
  - [j] Find the area of triangle  $PQR$ . (Do **NOT** use decimal approximations.)
  - [k] Find  $\angle QRP$ . (Round your answer to 2 decimal places.)
  - [l] Find the general equation of the plane passing through  $P$ ,  $Q$  and  $R$ .
  - [m] Find parametric equations for the line which passes through  $P$  and is also parallel to  $\vec{u}$ .
  - [n] Find symmetric equations for the line which passes through  $Q$  and is also perpendicular to the plane  $-2x - 3y + z = 9$ .
  - [o] Find the equation of the sphere with  $P$  and  $Q$  as endpoints of a diameter.
  - [p] Find the volume of the parallelepiped with  $\vec{u}$ ,  $\vec{w}$  and  $\vec{t}$  as adjacent edges.
- [3] Which octant or octants contain all points  $(x, y, z)$  where  $xz < 0$ ?
- [4] Consider the sphere  $x^2 + y^2 + z^2 - 4x + 6y + 10z + 29 = 0$ .
- [a] Find the center and radius of the sphere.
  - [a] Find the equations of the  $xy$ -,  $xz$ - and  $yz$ -traces of the sphere, and describe each trace.
- [5] Write vectors  $\vec{d}$ ,  $\vec{e}$  and  $\vec{f}$  in terms of vectors  $\vec{a}$ ,  $\vec{b}$  and  $\vec{c}$  in the diagram on the right.



## Math 43 Midterm 2 Review Answers

- [1] [a]  $\approx 2.94$  radians  
[b]  $\langle -\frac{1}{\sqrt{14}}, \frac{3}{\sqrt{14}}, \frac{2}{\sqrt{14}} \rangle$  *or*  $\langle \frac{1}{\sqrt{14}}, -\frac{3}{\sqrt{14}}, -\frac{2}{\sqrt{14}} \rangle$   
[c]  $(-6, 7, -12)$   
[d]  $a = \frac{5}{4}, \quad b = \frac{15}{4}$   
[e]  $-9$
- [2] [a] octant 6  
[b]  $(-9, 4, 1)$   
[c]  $\langle 6, -2, 1 \rangle$   
[d]  $2\vec{i} + 6\vec{j} - 5\vec{k}$   
[e]  $\sqrt{65}$   
[f]  $\langle -\frac{2}{\sqrt{65}}, -\frac{6}{\sqrt{65}}, \frac{5}{\sqrt{65}} \rangle$   
[g]  $\langle \frac{36}{\sqrt{41}}, -\frac{12}{\sqrt{41}}, \frac{6}{\sqrt{41}} \rangle$   
[h]  $\approx -7.99$   
[i]  $\approx 17.47$   
[j]  $2\sqrt{165}$   
[k]  $\approx 1.47$  radians  
[l]  $x + 8y + 10z - 9 = 0$   
[m]  $x = -5 + 6t, \quad y = -2 - 2t, \quad z = 3 + t$  **OTHER ANSWERS POSSIBLE**  
[n]  $\frac{x-3}{2} = \frac{y-2}{3} = -z-1$  **OTHER ANSWERS POSSIBLE**  
[o]  $(x+1)^2 + y^2 + (z-1)^2 = 24$   
[p]  $28$
- [3] octants 2, 3, 5, 8
- [4] [a] center  $= (2, -3, -5)$ , radius  $= 3$   
[b] no  $xy$ -trace  
 $xz$ -trace is point  $(2, 0, -5)$   
 $yz$ -trace has equation  $(y+3)^2 + (z+5)^2 = 5$  [circle in  $yz$ -plane, center  $= (0, -3, -5)$ , radius  $= \sqrt{5}$ ]
- [5]  $\vec{d} = \vec{a} + \vec{b}, \quad \vec{e} = \vec{c} - \vec{b}, \quad \vec{f} = -\vec{a} - \vec{c}$